▸ LEONARDO CERAGIOLI, FRANCESCO MONTESI, AND ANTONIO PICCOLOMINI D'ARAGONA, *A survey of two strands in proof-theoretic semantics.*
Department of Philosophy, University of Milan, Via Festa del Perdono 7, Italy.
E-mail: leonardo.ceragioli@unimi.it.
DSU, "Federico II" University of Naples, via Porta di Massa 1, Italy.
E-mail: francesco.montesi@unina.it.
CFvW-Center, Eberhard Karls Universität Tübingen, Doblerstraße 33, Germany.
E-mail: antonio.piccolomini-daragona@uni-tuebingen.de.

Since Schroeder-Heister introduced the term "proof-theoretic semantics" (PTS) in 1991 [82], many diverse ideas and results have been said to belong to the PTS field. Also, what Schroeder-Heister's expression refers to was already there well before 1991. The birth of PTS traces back to the early 1970s, with papers by Prawitz [65, 66], or maybe to the 1930s, with Gentzen's seminal works [30, 31].

In our talk, we focus on PTS as it is *currently* conceived, and try to provide an overview of the most relevant steps and achievements in the development of two distinct strands in today's researches in the field. We call these the *From Semantics to Rules* (SR) and *From Rules to Semantics* (RS) approach, respectively.

We do not pretend that SR plus RS amounts to the full PTS field, nor that SR and RS are completely kept apart from each other. Still, it seems to us that these two perspectives are not often, or not clearly distinguished, and that this may lead to confusion on what PTS is, or should be expected to do. Besides focusing on the differences between SR and RS, however, we also insist on some principles they share.

One of these principles is Gentzen's tenet that logical meaning is determined by introduction rules in Natural Deduction, while elimination rules can be somehow shown to be consequences of this semantic determination—or unique functions of the privileged proof-principles [31]. The differences between SR and RS stem from how the Gentzenian principle is understood, from the role it plays with accompanying assumptions in the given overall approach, and from the purposes it is taken to serve.

In the SR framework, Gentzen's tenet is subsumed to a notion of *global* validity of proof-structures, i.e., derivation-trees in the Natural Deduction format. A proof-structure is valid if it reduces to one which ends by introduction. The reduction procedure results from proof-rewriting functions associated to non-introduction rules—approaches which prioritise elimination rules are available too [35, 53, 78, 95], but we leave them aside. The validity of a non-introduction rule is *not* given by its proof-theoretic balance over the introduction rules, but by the fact that it preserves the validity of the proof-structures it gives rise to [50, 83]. This has three important consequences.

First, a non-introduction rule needs not be associated to just *one* rewriting function, as it may require *no* rewriting function, or *many* rewriting functions—even for rules others than that at issue. Second, the rule might not show *any* balance relative to the introduction rules. Third, validity of a proof-structure is *not* equivalent to *normalisability* of a Natural Deduction derivation, so usual counterexamples to validity of rules based on non-normalisability—e.g., Read's *bullet* [69]—might be of *no use* to show that given rules are invalid in the SR sense.

SR looks at PTS as a semantics which, while proof-theoretic in spirit, is "operationally" similar to model-theoretic or Kripkean semantics: given a formal language $\mathscr{L}$, one defines a proof-based notion of consequence for (sets of) formulas of $\mathscr{L}$, and then one proves standard results for given logics Σ over $\mathscr{L}$ relative to the consequence relation at issue (one also provides "models" for the non-logical dictionary of $\mathscr{L}$ in terms of atomic proof-systems). To attain the intended generality, hence, the consequence relation cannot coincide with that of derivability in some specific Σ. The question is

not whether or how the rules of Σ give meaning to the logical dictionary of $\mathscr{L}$. Rather, the task is giving validity criteria for *arbitrary* non-introduction rules on $\mathscr{L}$, even in the presence of a Σ which is proof-theoretically well-behaved.

In the SR tradition, hence, the research has focused on such topics as functional completeness and categoricity of connectives [67, 79, 81, 84], (Prawitz's) completeness (conjecture) [4, 15, 55, 56, 57, 58, 59, 60, 63, 77, 88, 89, 90], theory of atomic proof-systems [61, 62], criteria of acceptability of reductions and of identity of proofs [14, 101, 103]—possibly in connection with λ-calculus [1, 3]—and (completeness-related) applications to bilateralism [2, 5], bunched implication [39, 34], category theory [68], ecumenic logic [6], linear logic [7, 9, 37, 38], modal logic [23], negation-as-failure [32], phase semantics [96], and reductive logic [33, 36].

In the RS framework, Gentzen's tenet is taken *literally* into account, so that the main issue becomes which *criteria* rules must satisfy to confer logical meaning. Instead of global validity of proof-structures, hence, the RS tradition focuses on the local and analytic validity of inference rules, imposing restrictions on the shape of both introduction and elimination rules.

The *criteria* for the introduction rules are usually less controversial—but see [22, 49, 69]—so we focus on the elimination rules only. The justification of the latter is sorted out in different ways: availability of local reduction steps (one for each rule) [12, 13], criteria for their general form (general-elimination harmony) [28, 69, 72], or considerations of proof-theoretic strength of rules and formulas [91, 98, 100]. The shared aim is proving that given elimination rules are neither too weak nor too strong with respect to the deductive import of the corresponding introduction rules.

The focus on rules as meaning-conferring led to broad investigations into various formalisations of deduction, such as multiple-conclusion systems [69, 74], sequent-calculi [99], rules introducing connectives in subordinate positions [11, 48] and labelled calculi [71]. Each of these extensions is controversial and contested by other logicians working in the same framework—see, e.g., [76] for sequents, [25, 93] for multiple conclusions and subordinate positions, and [64] for labels.

This approach fades the distinction between PTS strictly understood and the wider field of inferentialism influenced by structural proof theory, sometimes making it hard to establish a clear non-arbitrary boundary [52, 100]. The connections with inferentialist theories of meaning also stimulated the application of RS methodologies for studying fragments of natural language [26, 27]. Moreover, the eventuality that more than one system of logic satisfies the criteria of analytic justification is sometimes discussed in the RS tradition, with both pluralist [42, 44, 75], and strictly monist [70] conclusions.

Other topics which have been investigated in the RS tradition are: harmony for classical logic [48, 51, 69]; harmony and conservative extension [10, 41, 86, 92, 98]; distinction between flattened [22, 28] and higher-level [73, 80] general-elimination harmony; completeness of the proof-theoretic characterization of negation [45, 46]; acceptability of bilateral systems [24, 26, 29, 46, 47, 76, 87]; debate on how to define strength of rules and formulas [16, 100]; harmony and structural rules [40, 41, 54, 98, 99]; proof-theoretic treatment of paradoxes [85, 97, 101, 102, 103].

Beside Gentzen's tenet, both the PTS trends we identify hark back to Dummett's philosophy. Dummett's general interests lie in the study of meaning broadly conceived. However, his investigations into the *theory of meaning*, i.e., the inquiry into the general principles and form that a systematic study of language should adopt [21], and into *meaning-theories*, i.e., the construction of such a theory for a given language [20], as well as into the nature and justification of deduction [19], can be also said to provide or inspire many of the relevant notions of PTS. We concentrate on two specific concepts—and on their many facets [17, 43, 94]: *harmony*—that we already mentioned—and the

fundamental assumption.

Harmony—balance of two aspects of the use sentences, i.e., asserting and drawing consequences—is as known interpreted by Dummett either as a global requirement of *total harmony* in terms of conservativeness [8], or as a local requirement of *intrinsic harmony* for a set of rules of a particular logical constant—complemented by a symmetrical counterpart called *stability*. These are what allows for the systematicity of the analysis of meaning itself, but also for the possibility of critique and revision of the use of language [18], and for the justifiability of semantic systems [20]. Correspondingly, the assumption that, for any valid argument for a complex sentence, we can construct a valid argument for it which ends with an application of an introduction, grounds both the proof-theoretic justification procedure, and the meaning-theoretical account of the molecular character of language.

In the analysis of both sides of such concepts, Dummett sensibly relies upon insights from structural proof-theory, based on which he outlines a notion of argumental validity much in the spirit of the SR one, while insisting on the inherent semantic character of proof-theoretic balance, thus coming close to the RS approach instead. So, in Dummett's philosophy, aspects which were later picked up separately, appear instead as harmoniously articulated, and constitute a sort of "neutral and natural" field for the evaluation and assessment of the differences and similarities between the SR and the RS approach, respectively.[1]

[1] Sara Ayhan, *What are acceptable reductions? Perspectives from proof-theoretic semantics and type theory*, ***Australasian journal of logic***, vol. 20 (2023), no. 3, pp. 412–428.

[2] ——— *Meaning and identify of proofs in a bilateralist setting: a two-sorted typed lambda-calculus for proofs and refutations*, ***Journal of logic and computation***, vol. 35 (2025), no. 2.

[3] Sara Ayhan and Heinrich Wansing, *On synonymy in proof-theoretic semantics: the case of* `2Int`, ***Bulletin of the section of logic***, vol. 52 (2023), no. 2, pp. 187–237.

[4] Victor Barroso Nascimento, ***Foundational studies in proof-theoretic semantics***, PhD Dissertation, Rio de Janeiro State University (UERJ), http://www.bdtd.uerj.br/handle/1/22473, (2024).

[5] Victor Barroso Nascimento and Maria Osório *Bilateral base-extension semantics*, https://arxiv.org/abs/2505.01593, (2025).

[6] Victor Barroso Nascimento, Luiz Carlos Pereira and Elaine Pimentel, *An ecumenical view of proof-theoretic semantics*, https://arxiv.org/abs/2306.03656, (2023).

[7] Victor Barroso Nascimento, Ekaterina Piotrovskaya and Elaine Pimentel, *A proof-theoretic approach to the semantics of classical linear logic*, https://arxiv.org/abs/2504.08349, (2025).

[8] Nuel Belnap, *Tonk, plonk and plink*, ***Analysis***, vol. 24 (1962), no. 6, pp. 130–134.

[9] Yll Buzoku, *A proof-theoretic semantics for intuitionistic linear logic*, https://arxiv.org/abs/2402.01982, (2024).

[10] Leonardo Ceragioli, *Peano's counterexample to harmony*, ***Theoria***, vol. 85 (2019), no. 6, pp. 459–484.

[1] Leonardo Ceragioli was funded by the Ministero dell'Università e della Ricerca (MUR) through PRIN 2022 Project SMARTEST – Simulation of Probabilistic Systems for the Age of the Digital Twin (20223E8Y4X). Antonio Piccolomini d'Aragona was funded by the grant PI 1965/1-1 for the DFG project *Revolutions and paradigms in logic. The case of proof-theoretic semantics.*

[11] ——— *Single-assumption systems in proof-theoretic semantics*, ***Journal of philosophical logic***, vol. 51 (2022), pp. 1019–1054.

[12] Rowan Davies and Frank Pfenning, *A modal analysis of staged computation*, ***Journal of the ACM***, vol. 48 (2001), no. 3, pp. 555–604.

[13] ——— *A judgmental reconstruction of modal logic*, ***Mathematical Structures in Computer Science***, vol. 11 (2001), no. 4, pp. 511–540.

[14] Wagner de Campos Sanz and Hermógenes Oliveira, *On Dummett's verificationist justification procedure*, ***Synthese***, vol. 193 (2016), pp. 2539–2559.

[15] Wagner de Campos Sanz, Thomas Piecha and Peter Schroeder-Heister, *Constructive semantics, admissibility of rules and the validity of Peirce's law*, ***Logic journal of the IGPL***, vol. 22 (2013), no. 2, pp. 297–308.

[16] Pedro del Valle-Inclán, *Proof-theoretic harmony and the strength of rules*, ***Erkenntnis***, (2025).

[17] Bogdan Dicher, *Weak disharmony: some lessons for proof.theoretic disharmony*, ***Review of symbolic logic***, vol. 9 (2016), no. 3, pp. 583–602.

[18] Michael Dummett, ***Frege. Philosophy of language***, Duckworth, 1973.

[19] ——— *The justification of deduction*, ***Truth and other enigmas***, Harward University Press, 1978, pp. 290-318.

[20] ——— ***The logical basis of metaphysics***, Duckworth, 1991.

[21] ——— *What is a theory of meaning? (II)*, ***The seas of language***, Clarendon Press, 1993, pp. 34–93.

[22] Roy Dyckhoff, *Some remarks on proof-theoretic semantics*, ***Advances in proof-theoretic semantics*** (T. Piecha and P. Schroeder-Heister, editors), Springer, 2016, pp. 79–93.

[23] Timo Eckhardt and David Pym, *Base-extension semantics for modal logic*, ***Logic journal of the IGPL***, vol. 33 (2025), no. 2.

[24] Fernando Ferreira, *The co-ordination principles: a problem for bilateralism*, ***Mind***, vol. 117 (2008), pp. 1051–1057.

[25] Camillo Fiore, *A Structural Tonk*, ***Analysis***, vol. 84 (2023), no. 1, pp. 13–22.

[26] Nissim Francez, ***Proof-theoretic semantics***, College Publications, 2015.

[27] Nissim Francez and Roy Dyckhoff, *Proof-theoretic semantics for a natural language fragment*, ***Linguistics and Philosophy***, vol. 33 (2010), pp. 447–477.

[28] ——— *A Note on Harmony*, ***Journal of Philosophical Logic***, vol. 41 (2012), no. 3, pp. 613–628.

[29] Michael Gabbay, *Bilateralism does not provide a proof theoretic treatment of classical logic (for technical reasons)*, ***Journal of applied logic***, vol. 25 (2017), pp. S108–S122.

[30] Gerhard Gentzen, *Über die Existenz unabhängiger Axiomensysteme zu unendlichen Satzsystemen*, ***Mathematische Annalen***, vol. 107 (1932), pp. 329-.350.

[31] ——— *Untersuchungen über das logische Schließen I, II*, ***Mathematische Zeitschrift***, vol. 39 (1935), pp. 176–210, 405–431.

[32] Alexander V. Gheorghiu and David Pym, *Definite formulae, negation-as-failure, and the base-extension semantics of intuitionistic propositional logic*, ***Bulletin of the section of logic***, vol. 52 (2023), no. 2, pp. 239–266.

[33] ——— *Reductive logic, proof-search, and coalgebra: a perspective from resource semantics*, ***Samson Abramsky on logic and structure in computer science and beyond***, (A. Palmigiano and M. Sadrzadeh, editors), Springer, 2023.

[34] ——— *Semantical analysis of the logic of bunched implications*, ***Studia Logica***, vol. 111 (2023), pp. 525–571.

[35] ——— *From proof-theoretic validity to base-extension semantics for intuitionistic propositional logic*, ***Studia Logica***, (2025).

[36] ——— *Semantic foundations of deductive reasoning*, ***Topoi***, (2025).

[37] Alexander V. Gheorghiu, Tao Gu and David Pym, *Proof-theoretic semantics for intuitionistic multiplicative linear logic*, ***Automated reasoning with analytic tableaux and related methods. Tableaux 2023***, (R. Ramanayake and J. Urban, editors), Lecture notes in computer science, 2023, pp. 367–285.

[38] ——— *Proof-theoretic semantics for intuitionistic multiplicative linear logic*, ***Studia Logica***, (2024).

[39] ——— *Proof-theoretic semantics for the logic of bunched implications*, ***Studia Logica***, (2025)

[40] Ian Hacking, *What is logic?*, ***The journal of philosophy***, vol. 76 (1979), no. 6, pp. 285–319.

[41] Ole Hjortland and Shawn Standefer, *Inferentialism, structure and conservativeness*, ***From rules to meanings: new essays on inferentialism*** (O. Beran, V. Kolman and L. Koreň, editors), Routledge, New York, 2018, pp. 115–140.

[42] Nils Kürbis, *Pluralism and the logical Basis of metaphysics*, ***Logica Yearbook 2006*** (O. Tomala and R. Honzík, editors), College Publications, Prague, 2007, pp. 157–169.

[43] ——— *How fundamental is the fundamental assumption?*, ***Teorema: International Journal of Philosophy***, vol. 2 (2012), no. 2, pp. 5–19.

[44] ——— *Proof-theoretic semantics, a problem with negation and prospects for modality*, ***Journal of Philosophical Logic***, vol. 44 (2015), pp. 713–727.

[45] ——— *Some comments on Ian Rumfitt's bilateralism*, ***Journal of Philosophical Logic***, vol. 45 (2016), pp. 623–644.

[46] ——— ***Proof and falsity: A logical investigation***, Cambridge University Press, 2019.

[47] ——— *Supposition: A Problem for Bilateralism*, ***Bulletin of the Section of Logic***, vol. 52, no. 3, (2023), pp. 301–327.

[48] Peter Milne, *Harmony, purity, simplicity and a "seemingly magical fact"*, ***Monist***, vol. 85 (2002), no. 4, pp. 498–534.

[49] ——— *Inversion principles and introduction rules*, ***Dag Prawitz on proofs and meaning*** (H. Wansing, editor), Springer, 2015, pp. 189–224.

[50] Enrico Moriconi, ***Significato e dimostrazione. Michael Dummett, Dag Prawitz, Per Martin-Löf***, Franco Angeli, 1993.

[51] Julien Murzi, *Classical harmony and separability*, ***Erkenntnis***, vol. 85 (2020), no. 2, pp. 391–415.

[52] Julien Murzi and Florian Steinberger, *Inferentialism*, ***A Companion to the philosophy of language*** (C. Wright, B. Hale and A. Miller, editors), Wiley-Blackwell, Hoboken, New Jersey, 2017, pp. 197–224.

[53] Hermógenes Oliveira, *On Dummett's pragmatist justification procedure*, ***Erkenntnis***, vol. 86 (2021), pp. 429–455.

[54] Francesco Paoli, *Semantic Minimalism for Logical Constants*, ***Logique et Analyse***, vol. 57 (2014), no. 227, pp. 439–461.

[55] Antonio Piccolomini d'Aragona, *A note on schematic validity and completeness in Prawitz's semantics*, ***Current topics in logic and the philosophy of science. Papers from SILFS 2022 postgraduate conference***, (F. Bianchini, V. Fano and P. Graziani, editors) College Publications, 2024, pp. 143–158.

[56] ——— *A comparison of three kinds of monotonic proof-theoretic semantics and the base-incompleteness of intuitionistic logic*, https://arxiv.org/abs/2501.03297, (2025).

[57] ——— *Some results in non-monotonic proof-theoretic semantics*, ***Studia Logica***, (2025).

[58] ——— *Uniform validity of atomic Kreisel-Putnam rule in monotonic proof-theoretic semantics*, https://arxiv.org/abs/2503.19930, (2025).

[59] Thomas Piecha, *Completeness in proof-theoretic semantics*, ***Advances in proof-theoretic semantics***, (T. Piecha and P. Schroeder-Heister, editors), Springer, 2016, pp. 231–251.

[60] Thomas Piecha and Wagner de Campos Sanz, *A critical remark on the BHK interpretation of implication*, ***Philosophia Scientiae***, (2014), pp. 123–54.

[61] Thomas Piecha and Peter Schroeder-Heister, *Atomic systems in proof-theoretic semantics: two approaches*, ***Epistemology, knowledge and the impact of interaction***, (J. Redmond, and O. Pombo Martins and Á. Nepomuceno Fernández, editors), Springer, 2016, pp. 47–62.

[62] ——— *The definitional view of atomic proof systems in proof-theoretic semantics*, ***The Logica Yearbook 2016***, (P. Arazim and T. Lávička, editors), College Publications, 2017, pp. 185–200.

[63] Thomas Piecha, Wagner de Campos Sanz and Peter Schroeder-Heister, *Failure of completeness in proof-theoretic semantics*, ***Journal of philosophical logic***, vol. 44 (2015), pp. 321–335.

[64] Francesca Poggiolesi and Greg Restall, *Interpreting and applying proof theories for modal Logic*, ***New waves in philosophy*** (G. Restall and G. Russell, editors), Palgrave Macmillan, London, 2012, pp. 39–62.

[65] Dag Prawitz, *Ideas and results in proof theory*, ***Proceedings of the second Scandinavian logic symposium*** (J. E. Fenstad, editor), Elsevier, 1971, pp. 235–307.

[66] ——— *Towards a foundation of general proof theory*, ***Proceedings of the Fourth International Congress for Logic, Methodology and Philosophy of Science, Bucharest, 1971*** (P. Suppes, L. Henkin, A. Joja and G. C. Moisil, editor), Elsevier, 1973, pp. 225–250.

[67] ——— *Proofs and the meaning and completeness of the logical constants*, ***Essays on mathematical and philosophical logic***, (J. Hintikka, I. Niiniluoto and E. Saarinen, editors), 1979, pp. 25–40.

[68] David Pym, Eike Ritter and Edmund Robinson, *Categorical proof-theoretic semantics*, ***Studia Logica***, vol. 113 (2025), pp. 125–162.

[69] Stephen Read, *Harmony and Autonomy in Classical Logic*, ***Journal of Philosophical Logic***, vol. 29 (2000), pp. 123–154.

[70] ——— *Monism: the one true logic*, ***A Logical approach to philosophy: essays in memory of Graham Solomon*** (D. Devidi and T. Kenyon, editors), Springer, 2006, pp. 193–209.

[71] ——— *Harmony and modality*, ***Dialogues, logics and other strange things: essays in honour of Shahid Rahman*** (C. Dégremont, L. Kieff and H. Rückert, editors), College Publications, London, 2008, pp. 285–303.

[72] ——— *General-elimination harmony and the meaning of the logical constants*, ***Journal of philosophical logic***, vol. 39 (2010), pp. 557–576.

[73] ——— *General-elimination harmony and higher-level rules*, ***Dag Prawitz on proofs and meaning*** (H. Wansing, editor), Springer, 2015, pp. 293–312.

[74] Greg Restall, *Multiple Conclusions*, ***Logic, Methodology and Philosophy of Science: Proceedings of the Twelfth International Congress*** (Oviedo, Spain), (P. Hajek, L. Valdes-Villanueva and D. Westerstahl, editors), Kings' College Publications, 2005, pp. 189–205.

[75] ——— *Pluralism and proofs*, ***Erkenntnis***, vol. 79 (2014), no. 2, pp. 279–291.

[76] Ian Rumfitt, *'Yes' and 'no'*, ***Mind***, vol. 109 (2000), no. 436, pp. 781–823.

[77] Tor Sandqvist, *Classical logic without bivalence*, ***Analysis***, vol. 10969 (2009), no. 2, pp. 211–218.

[78] ——— *Base-extension semantics for intuitionistic sentential logic*, ***Logic journal of the IGPL***, vol. 23 (2015), no. 5, pp. 719–731.

[79] Peter Schroeder-Heister, *The completeness of intuitionistic logic with respect to a validity concept based on an inversion principle*, ***Journal of philosophical logic***, vol. 12 (1983), pp. 359–377.

[80] ——— *A Natural Extension of Natural Deduction*, ***Journal of symbolic logic***, vol. 49 (1984), no. 4, pp. 1284–1300.

[81] ——— *Generalized rules for quantifiers and the completeness of the intuitionistic operators* $\wedge$, $\vee$, $\rightarrow$, $\forall$, $\exists$, ***Proceedings of the Logic Colloquium. Held in Aachen, July 18-23, 1983***, (E. Börger, W. Oberschelp, M. M. Richter, B. Schinzel and W. Thomas, editors), 1984, pp. 399–426.

[82] ——— *Uniform proof-theoretic semantics for logical constants*, ***Journal of symbolic logic***, vol. 56 (1991), no. 3, pp. 1142.

[83] ——— *Validity concepts in proof-theoretic semantics*, ***Synthese***, vol. 148 (2006), pp. 525–571.

[84] ——— *Prawitz's completeness conjecture: a reassessment*, ***Theoria. A Swedish journal of philosophy***, vol. 5 (2024), no. 5, pp. 515–527.

[85] Peter Schroeder-Heister and Luca Tranchini, *How to Ekman a Crabbé-Tennant*, ***Synthese***, vol. 199 (2021), no. Suppl. 3, pp. 617–639.

[86] Steward Shapiro, *Induction and Indefinite Extensibility: The Gödel Sentence is True, but Did Someone Change the Subject?*, ***Mind***, vol. 107 (1998), no. 427, pp. 597–624.

[87] Ryan Simonelli, *Supposition: no Problem for Bilateralism*, ***Bulletin of the Section of Logic***, https://doi.org/10.18778/0138-0680.2025.01, (2025).

[88] Will Stafford, *Proof-theoretic semantics and inquisitive logic*, ***Journal of philosophical logic*** vol. 50 (2021), pp. 1199–1229.

[89] ——— *Proof-theoretic validity isn't intuitionistic; so what?*, ***Australasian journal of philosophy***, vol. 103 (2024), no. 2, pp. 356–372.

[90] Will Stafford and Victor Barroso Nascimento, *Following all the rules: intuitionistic completeness for generalised proof-theoretic validity*, ***Analysis*** vol. 83 (2023), no. 3, pp. 507–516.

[91] Florian Steinberger, *Not so stable*, ***Analysis***, vol. 69 (2009), no. 4, pp. 655–661.

[92] ——— *What Harmony Could and Could not Be*, ***Australasian journal of philosophy***, vol. 89 (2011), no. 4, pp. 617–639.

[93] ——— *Why conclusions should remain single*, ***Journal of philosophical logic***, vol. 40 (2011), no. 3, pp. 333–355.

[94] ——— *On the equivalence conjecture for proof-theoretic harmony*, ***Notre Dame journal of formal logic***, vol. 54 (2013), no. 1, pp. 79–86.

[95] Ryo Takemura, *Investigation of Prawitz's completeness conjecture in phase semantics framework*, ***Journal of humanities and sciences of Nihon University***, vol. 23 (2017), no. 1.

[96] ——— *A completeness theorem in proof-theoretic semantics via set-theoretic semantics*, https://arxiv.org/abs/2505.10765, (2025).

[97] Neil Tennant, *Proof and Paradox*, ***Dialectica***, vol. 36 (1982), no. 2-3, pp. 265–296.

[98] ——— ***The taming of the true***, Oxford University Press, 1997.

[99] ——— ***Core logic***, Oxford University Press, 2017.

[100] ——— *Inferentialism, logicism, harmony, and a counterpoint*, ***Essays for Crispin Wright: logic, language and mathematics*** (A. Miller, editor), Oxford University Press, Oxford, 2020, pp. 223–247.

[101] Luca Tranchini, *Proof-theoretic semantics, paradoxes and the distinction between sense and denotation*, ***Journal of logic and computation***, vol. 26 (2016), no. 2, pp. 495–512.

[102] ——— *Proof, meaning and paradox: some remarks*, ***Topoi***, vol. 38 (2019), pp. 591–603.

[103] ——— ***Harmony and paradox. Intensional aspects of proof-theoretic semantics***, Springer, 2024.