▶ ANTON FERNÁNDEZ, *A classification of the* KPl*-provably total set-recursive functions.*

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We present a result classifying the total set-recursive functions of the set theory KPl and of the two weaker versions $\mathsf{KPl}^{\mathsf{r}}$ and W-KPl. We also show that, fixing the length of the proof and restricting the provably total functions accordingly, we can prove within the theories the conclusions of the respective classification theorems.

The fragment KP, for Kripke-Platek, of ZFC is obtained by dropping the Power-Set Axiom and restricting the schemes of Separation and Collection to Δ_0-formulas. A transitive set model of KP is called an admissible set and KPl, for Kripke-Platek-limit, stipulates that there are unboundedly many admissible sets in the universe, which is thus a limit of admissibles. We obtain $\mathsf{KPl}^{\mathsf{r}}$ and W-KPl by weakening the axiom of foundation in KPl to set-foundation and to full foundation restricted to the natural numbers respectively.

In [2], the authors classified the total Σ_1-definable set functions of KP. We extend this result to a classification of the total set-recursive functions of the three mentioned theories. We rephrase the theorem in terms of set-recursive functions because KPl proves the totality of some Σ_1-definable set functions that are not effectively describable, such as

$$a \mapsto \text{smallest admissible set containing } a.$$

Via Van de Wiele's theorem ([3]), set-recursion amounts to the fact that the functions we consider are Σ_1-definable with the same formula in any admissible set. Our results roughly say that, for T among KPl, $\mathsf{KPl}^{\mathsf{r}}$ and W-KPl, given such a function for which T proves its totality, for any set x the image $f(x)$ belongs to an initial segment $L_{G_n^T(x)}(x)$ of the constructible hierarchy relativized to x. Here, $G_n^T(x)$ is an ordinal-valued function bounded by the relativized Π_1^1-ordinal of T. To prove such classifications results, we make use of a relativized ordinal analysis of the theories in question. We build a recursive ordinal notation system for KPl based on [1] and define an infinitary proof system from this notation system. Then, we embed KPl into this system, where we prove some cut-elimination results. Finally, we extract the classification of the total set-recursive functions from a cut-free proof in the infinitary system. We show the results for $\mathsf{KPl}^{\mathsf{r}}$ and W-KPl similarly.

Lastly, we show that the functions $G_n^{\mathsf{KPl}}(x)$ are in fact an optimal bound (similarly for $\mathsf{KPl}^{\mathsf{r}}$ and W-KPl). We show that, provably in KPl, $G_n^{\mathsf{KPl}}(x)$ is well-founded for all x, leading to the aforementioned strengthening of the classification theorem.

This is joint work with Juan P. Aguilera and Joost J. Joosten.

[1] W. Buchholz, *A new system of proof-theoretic ordinal functions*, ***Annals of Pure and Applied Logic***, vol. 32 (1986), pp. 195–207.

[2] J. Cook and M. Rathjen, *Classifying the total set functions of Kp and Kp(P)*, ***Journal of Logics and their applications***, vol. 3 (2016), no. 4, pp. 681–753.

[3] J. Van de Wiele, *Recursive dilators and generalized recursions*, ***Proceedings pf the Herbrand symposium***, vol. 107 (1982), pp. 325–332.