A Modular Ordinal Analysis for Fragments of Induction Principles

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Modular ordinal analysis, presented by Dieter Probst in his habilitationsschrift [5], aims to compute proof-theoretic invariants of a theory by decomposing the theory into some modules. It enables us to compute the invariants of a theory from that of a weaker one. Recently, Fedor Pakhomov and James Walsh [3, 4] proposed a new approach to compute invariants by iterated reflection principles, and Aguilera and Pakhomov [1] defined new higher invariants of theories in terms of Girard's dilators and ptykes. We provide an idea that makes it possible to understand relations between mathematical theories and to find a ray of hope for an ordinal analysis for full second order arithmetic Z_2. As a pilot study, we calculate usual and higher proof-theoretic invariants of syntactic reflection principles Π^1_{n+1}-$\mathsf{RFN}(T)$ and, by combing Frittation's result [2], fragments of second-order induction principles $T + \mathsf{I}\Pi^1_{n+1}$ for a reasonable theory T.

References

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