▶ ALI ENAYAT, *Tarski-style axiomatic truth theories.*

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The origins of the area known as Formal/Axiomatic Theories of Truth lie in Tarski's explication of the notion of 'truth in a structure', and his famous theorem about undefinability of truth. Over the past decades, substantive contributions of a number of logicians – of both mathematical and philosophical persuasions – have enriched the subject's literature.

In this tutorial I will focus on *Tarski-style axiomatic truth theories over arithmetic and set theory*, with an emphasis on the use of model-theoretic techniques for extracting conservativity, interpretability, and other proof-theoretic information.

The first lecture presents an overview of the general concepts, questions, techniques, and results of the subject, with emphasis on truth theories over PA (Peano Arithmetic). We will see the proof outline of the model-theoretic proof of *conservativity* of the truth theory known as $\mathsf{CT}^-[\mathsf{PA}]$ (an extension of PA with a compositional truth predicate, the minus superscript indicates that there is no additional induction in the extended language). The proof methodology will be shown to be quite versatile and capable of establishing many other conservativity results. Moreover, it lends itself to appropriate arithmetizations that yield the *interpretability* of $\mathsf{CT}^-[\mathsf{PA}]$ in PA, and the *feasible proof-theoretic reducibility* of $\mathsf{CT}^-[\mathsf{PA}]$ to PA.

The second lecture concerns the 'many faces theorem' of the canonical extension $\mathsf{CT}_0[\mathsf{PA}]$ of $\mathsf{CT}^-[\mathsf{PA}]$, which includes a modicum of induction, namely Δ_0-induction for formulae involving the truth predicate. This theorem suggests that $\mathsf{CT}_0[\mathsf{PA}]$ is the weakest theory among 'natural' nonconservative extensions of $\mathsf{CT}^-[\mathsf{PA}]$. We will also explore the intimate relationship between $\mathsf{CT}_0[\mathsf{PA}]$ and a well-known strengthening of the fragment ACA_0 of second order arithmetic.

The focus of the third lecture is on truth theories over ZF (Zermelo–Fraenkel set theory). We will first go through some old and new conservativity results concerning $\mathsf{CT}^-[\mathsf{ZF}]$, and will then we examine certain nonconservative extensions of $\mathsf{CT}^-[\mathsf{ZF}]$ that emerge from the search for the set-theoretical counterpart of $\mathsf{CT}_0[\mathsf{PA}]$.